Level – 1	Daily Tutorial Sheet -2

16.(B) $f(x) = 1 + 3^x \log 3$ $\quad\therefore\quad F(x) = \int (1 + 3^x \log 3)\, dx = x + 3^x + c$

Since $F(2) = 2 + 9 + c = 7 \Rightarrow c = -4$

$\therefore\quad F(x) = 0 \Rightarrow x + 3^x - 4 = 0 \Rightarrow x + 3^x = 4$; which is true for $x = 1$

17.(C) We have $f(x) = \int \frac{(x^2 + \sin^2 x)}{1 + x^2} \sec^2 x\, dx = \int \frac{x^2 + (1 - \cos^2 x)}{1 + x^2} . \sec^2 x\, dx = \int \left(\sec^2 x - \frac{1}{1 + x^2} \right) dx$

Thus, $f(x) = \tan x - \tan^{-1} x + C$

As $f(0) = 0, C = 0 \Rightarrow f(x) = \tan x - \tan^{-1} x$ $\qquad$ Hence, $f(1) = \tan 1 - \tan^{-1} 1 = \tan 1 - \frac{\pi}{4}$

18.(A) $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx = \int \frac{(\sin^4 x - \cos^4 x)(\sin^4 x + \cos^4 x)}{(1 - 2\sin^2 x \cos^2 x)} dx$

$= \int \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{(1 - 2\sin^2 x \cos^2 x)} \times \left[(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x \right] dx$

$= \int \frac{\cos 2x . (1 - 2\sin^2 x \cos^2 x)}{(1 - 2\sin^2 x \cos^2 x)} dx = -\int \cos 2x\, dx = \frac{-1}{2} \sin 2x + B \quad \Rightarrow \quad A = \frac{-1}{2}$

19.(B) $\int \frac{1}{1 + (\log x)^2} d(\log x) = \int \frac{dt}{1 + t^2}$ $\qquad$ [Putting $\log x = t \Rightarrow d(\log x) = dt$] $= \tan^{-1} t + C = \tan^{-1}(\log x) + C$.

20.(C) We have,

$y = \int x^3 \cos x^4 dx = \frac{1}{4} \int \cos t\, dt$ $\qquad$ $\left[\text{Putting } x^4 = t \quad \Rightarrow \quad x^3 dx = \frac{1}{4} dt \right]$

$= \frac{1}{4} \sin t + C = \frac{1}{4} \sin x^4 + C$

Since the curve passes through (0, 0).

$\therefore\quad C = 0 \quad \therefore \quad y = \frac{1}{4} \sin x^4 \quad \Rightarrow \quad x^4 = \sin^{-1}(4y) \quad \Rightarrow \quad x = \sqrt[4]{\sin^{-1}(4y)}$

21.(A) We have, $f''(x) = \sec^2 x \quad \Rightarrow \quad f'(x) = \int \sec^2 x\, dx + C_1 = \tan x + C_1$

Since $f'(0) = 0 \quad \therefore \quad C_1 = 0 \quad \therefore \quad f'(x) = \tan x \quad \Rightarrow \quad f(x) = \int \tan x\, dx + C_2 = \log \sec x + C_2$

Since $f(0) = 0, \quad \therefore \quad C_2 = 0 \quad \therefore \quad f(x) = \log \sec x$.

22.(B) Let $f(x) = ax^3 + bx^2 + cx + d$

Since $f(0) = -1$ and $f(1) = 0$

$\Rightarrow \qquad d = -1$ and $a + b + c + d = 0 \Rightarrow d = -1$ and $a + b + c = 1$ $\qquad$ ...(i)

Since 0 is stationary point of $f(x)$, $\quad \therefore \quad f'(0) = 0 \Rightarrow 3a(0)^2 + 2b(0) + c = 0 \Rightarrow c = 0$

Since $f(x)$ does not have an extremum at $x = 0$

$\therefore \qquad f''(0) = 0 \Rightarrow b = 0$ and $\therefore$ from (i), $a = 1$

So, $f(x) = x^3 - 1 \qquad \therefore \qquad \int \frac{f(x)}{x^3 - 1} dx = \int 1\, dx = x + C$

23.(A) We have, $\int \frac{2^{1/x}}{x^2}\,dx = K.\,2^{1/x}$

Differentiating both sides w.r.t. x, we get $\frac{2^{1/x}}{x^2} = K\,.\,2^{1/x}\,.\left(\frac{-1}{x^2}\right).\log 2 \quad \Rightarrow \quad K = \frac{-1}{\log 2}$

24.(D) $\int \sec 2x\,dx = \frac{1}{2}\int \frac{2\sec 2x(\sec 2x + \tan 2x)}{(\sec 2x + \tan 2x)}dx = \frac{1}{2}\log|\sec 2x + \tan 2x| + C = \frac{1}{2}\log\left|\frac{1+\sin 2x}{\cos 2x}\right| + C$

$$= \frac{1}{2}\log\left|\frac{1+\cos\left(\frac{\pi}{2}-2x\right)}{\sin\left(\frac{\pi}{2}-2x\right)}\right| + C = \frac{1}{2}\log\left|\cot\left(\frac{\pi}{4}-x\right)\right| + C \quad \Rightarrow \quad f(x) = \frac{1}{2}\log|x| \text{ and } g(x) = \cot\left(\frac{\pi}{4}-x\right)$$

25.(B) Let $\int \frac{2^x}{\sqrt{1-4^x}}\,dx = \frac{1}{\log 2}\int \frac{1}{\sqrt{1-t^2}}\,dt$

Putting $2^x = t$, $2^x \log 2dx = dt$

$$I = \frac{1}{\log 2}\sin^{-1}\left(\frac{t}{1}\right) + C = \frac{1}{\log 2}\sin^{-1}(2^x) + C \qquad \therefore \qquad k = \frac{1}{\log 2},\ f(x) = 2^x$$

26.(C) $I = \int \frac{n\cos^{n-1}}{(1+\sin x)^n}dx = \int \frac{n\sec x dx}{(\sec x + \tan x)^n} = \int \frac{n(\sec x)(\sec x + \tan x)dx}{(\sec x + \tan x)^{n+1}}$

Put $(\sec x + \tan x) = t$ and $\sec x(\sec x + \tan x)dx = dt \Rightarrow I = \int \frac{ndt}{t^{n+1}} = \frac{-1}{t^n} = \frac{-1}{(\sec x + \tan x)^n} = \frac{-\cos^n x}{(1+\sin x)^n}$

27.(CD) $\int \tan 2x\,.\,\tan 3x\,.\,\tan 5x\;dx = \int(\tan 5x - \tan 2x - \tan 3x)dx$ [Using $\tan 5x = \tan(2x+3x)$]

$$= \frac{1}{5}\log|\sec 5x| - \frac{1}{2}\log|\sec 2x| - \frac{1}{3}\log|\sec 3x| + k = \log\left|\sec^{-1/2} 2x\,.\,\sec^{-1/3} 3x\,.\,\sec^{1/5} 5x\right| + k$$

$\Rightarrow \quad a = \frac{-1}{2},\ b = \frac{-1}{3}$ and $c = \frac{1}{5} \Rightarrow ab + bc + ca = 0$ and $a^{-3} + b^{-3} + c^{-3} = 3a^{-1}b^{-1}c^{-1}$

28.(D) Put $xe^x = t$, $(e^x + xe^x)dx = dt$ to get: $\int \frac{e^x(1+x)dx}{\cos^2(xe^x)} = \int \frac{dt}{\cos^2(t)} = \int \sec^2(t)dt = \tan(t) = \tan(xe^x) + c$

29.(B) $I = \int \frac{e^x dx}{(2+e^x)(e^x+1)}$. Put $e^x = t$ to get: $e^x dx = dt$

$$\Rightarrow \quad I = \int \frac{dt}{(t+2)(t+1)} = \int \frac{[(t+2)-(t+1)]dt}{(t+2)(t+1)} = \int\left(\frac{1}{t+1} - \frac{1}{t+2}\right)dt$$

$$= \ell n(t+1) - \ell n(t+2) + c = \ell n\left(\frac{t+1}{t+2}\right) + c = \ell n\left(\frac{e^x+1}{e^x+2}\right) + c$$

30.(CD) Let $1 - x^3 = t^2 \quad \Rightarrow \quad \int \frac{\sqrt{x}\,dx}{\sqrt{1-x^3}} = \int \frac{-2dt}{3x^{3/2}} = \int \frac{-2}{3}\frac{dt}{\sqrt{1-t^2}} = \frac{2}{3}\cos^{-1}(t) = \frac{2}{3}\cos^{-1}\left(\sqrt{1-x^3}\right)$

But $\cos^{-1}\sqrt{1-x^3}$ can also be written as $\sin^{-1}\left(x^{3/2}\right) \quad \Rightarrow \quad f(x) = \sin^{-1} x$ and $g(x) = x^{3/2} = x\sqrt{x}$